

R. F. BUCKET AREA

Booster Technical Note

No. 31

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In order to optimize the Booster Magnet Cycle the magnet field versus time function must be determined. If the field rises too fast the r.f. bucket area is reduced. Conversely, if the field rises too slowly too much time is wasted in a very crowded time cycle. A key input to this optimization process is the magnet field versus time function that keeps the r.f. accelerating bucket area constant.

I have computed this function using the following 2,3.

$$\text{r.f. Bucket area} = \frac{8C}{\pi ch} \alpha(\phi s) \left[\frac{eV E_0 \gamma}{2\pi Mh} \right]^{1/2}$$

where: $\beta^2 = 1 - \frac{E_0^2}{E^2}$

E_0 - rest energy of protons

$\gamma = E/E_0$ T - Kinetic energy

$E = T + E_0$

V - r.f. crest voltage

M - ratio - mass/charge

h - harmonic No.

v_h - horizontal nu (4.82)

ϕ_s - stable phase angle

$\alpha(\phi s)$ - ratio - see Table I

C - machine circumference

c - velocity of light

e - electronic charge

Using a selected r.f. bucket area the computing process proceeds as follows:

1. For a set of machine inputs the desired value of $\alpha(\phi_s)$ was determined.
2. From the value of $\alpha(\phi_s)$ the stable phase angle ϕ_s is obtained.
3. From the stable phase angle the energy gain per turn is computed.
4. From the energy gain per turn (per unit of time) the rate of magnet field rise is computed.
5. This process is iterated to generate the field-time function.
2.4 millisec. Steps were used in this integration.

The results are plotted in figures 1 and 2 for r.f. accelerating crest voltages of 44 and 48KV respectively. These figures also contain the initial and final dB/dT values for chosen r.f. bucket areas.

Figure 1 contains a dashed curve which represents a constant $(dB/dT)B$ curve which has been passed through the beginning and end points of the area equal leV-sec. curve. This function has sometimes been used to estimate the constant bucket area function when this information is missing. But, as one can see the approximation is not good. The correct curve starts with a smaller slope but finished with a higher slope. Unfortunately, more power is required to follow the correct curve than is required to follow the approximation.

References:

- 1) Bovet, Gouiran, Gumowski and Reich A Selection of Formulae and Data Useful for the Design of the A.G. Synchrotrons, Cern/MPS-SI/Int. D1/70/4 23 April, 1970.
- 2) Cole and Morton, Area and Bunching Factors of Partially Filled Buckets, UCID 10130 AS/Theoretical/02 Sept 21, 1984; LRL, Univ. of California.
- 3) RUGGIERO and Young, Booster r.f. Program for Heavy Ions, RHIC-AP-17, Brookhaven National Lab. May 31, 1985.

Table I (1)

RF "BUCKET" WIDTH, NORMALISED (HALF) HEIGHT AND AREA

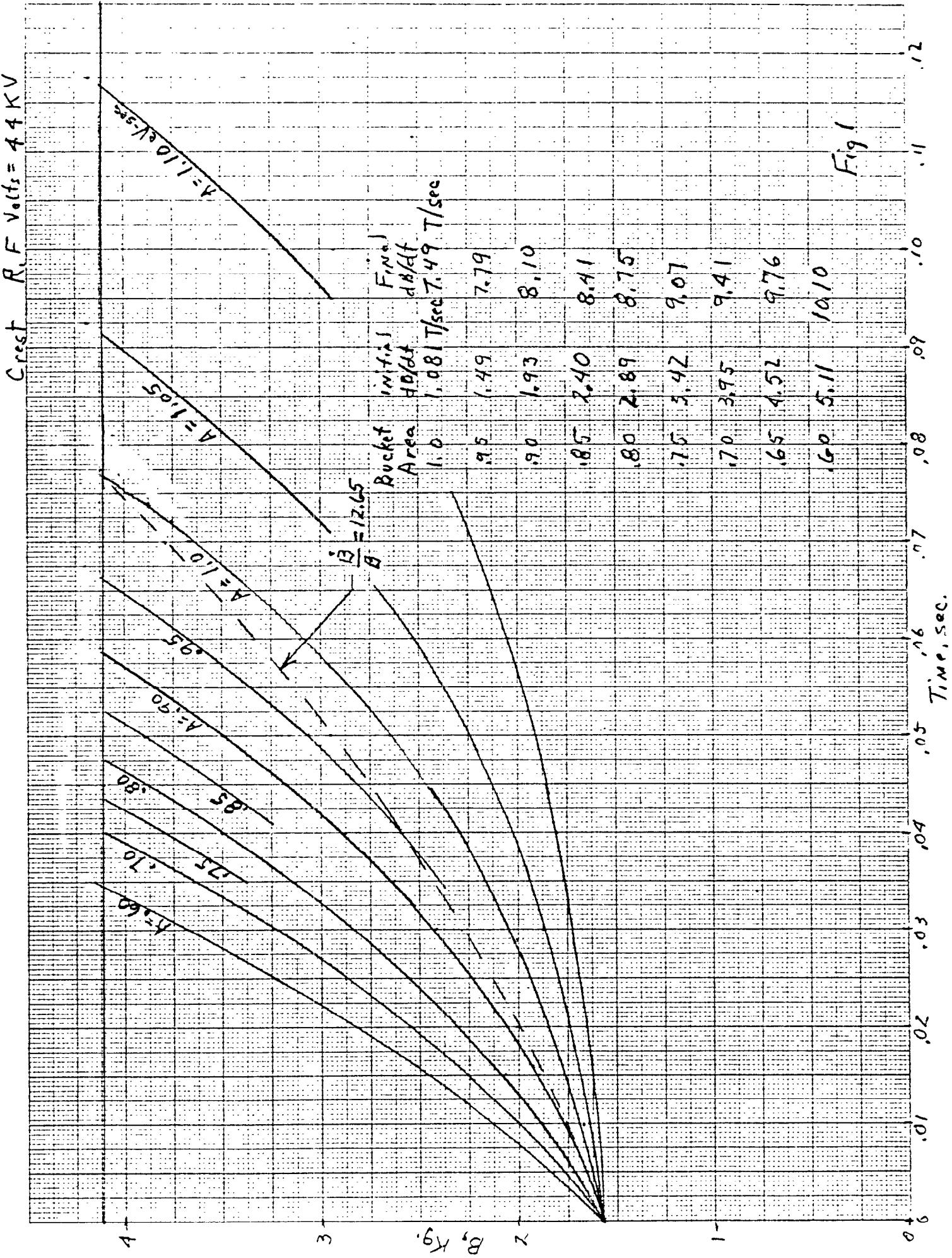
Stable phase		'Bucket' width		Half height		Area	
Ψ_B	Γ	Ψ_1	Ψ_2	$\Delta\varphi$	$\gamma(\Gamma)$	$\beta = \frac{\Delta E(\Gamma)}{\Delta E(0)}$	$\alpha = \frac{A(\Gamma)}{A(0)}$
0	0.00000	-180.0	180	360	1-414214	1-000000	1-000000
1	-017452	-154.0	179	333	1-394803	986275	954105
2	-034869	-143.5	178	321	1-375547	972517	917558
3	-052336	-135.5	177	312	1-355847	958729	884511
4	-069756	-128.8	176	305	1-336309	944913	853741
5	-087156	-122.9	175	298	1-316736	931073	824676
6	-104526	-117.6	174	292	1-297132	912111	796983
7	-121069	-112.6	173	286	1-277500	903229	770443
8	-139173	-108.1	172	280	1-257846	889431	744906
9	-156434	-103.7	171	275	1-238171	875519	720257
10	-173640	-99.6	170	270	1-218482	861597	696413
11	-1908C9	-95.7	169	265	1-198781	847666	673033
12	-207912	-92.0	168	260	1-179072	833130	650875
13	-224951	-88.4	167	255	1-159360	819191	629082
14	-241922	-84.9	166	251	1-139648	805853	607888
15	-258819	-81.5	165	247	1-119940	791917	587261
16	-275637	-78.2	164	242	1-100240	777987	567174
17	-292372	-75.0	163	238	1-080552	764066	547603
18	-309017	-71.9	162	234	1-060881	750156	528529
19	-325568	-68.9	161	230	1-041230	736261	509933
20	-342020	-65.9	160	226	1-021603	722282	491799
21	-358368	-63.0	159	222	1-002004	708524	471114
22	-3746C7	-60.1	158	218	982438	694688	456865
23	-390731	-57.3	157	214	962907	680818	440040
24	-406737	-54.5	156	210	943418	667097	423630
25	-422618	-51.8	155	207	923972	653347	407624
26	-438371	-49.1	154	203	904576	639632	392015
27	-453990	-46.4	153	199	885232	623954	376794
28	-469472	-43.8	152	196	865945	612316	361955
29	-484610	-41.2	151	192	846719	598721	347489
30	-500000	-38.7	150	189	827559	585172	333392
31	-515030	-36.2	149	185	808467	571673	319657
32	-522919	-33.7	148	182	789450	550225	306227
33	-544639	-31.2	147	178	770510	544433	293252
34	-559193	-28.6	146	175	751653	531499	280571
35	-573576	-26.3	145	171	732882	518226	268231
36	-587785	-23.9	144	168	714202	505017	256229
37	-601815	-21.6	143	165	695618	491076	244560
38	-615661	-19.2	142	161	677132	478805	233218
39	-629320	-16.9	141	158	658751	465807	222202

Stable phase		'Bucket' width		Half height		Area	
Ψ_B	Γ	Ψ_1	Ψ_2	$\Delta\varphi$	$\gamma(\Gamma)$	$\beta = \frac{\Delta E(\Gamma)}{\Delta E(0)}$	$\alpha = \frac{A(\Gamma)}{A(0)}$
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(From Ref. 14; note that in this reference $\gamma(0) = \Phi_{\max}(0)[\sqrt{2}2\kappa v_o(0)] = \sqrt{2}$ – rather than 2 – for computational convenience. See page 31 for other definitions and a graphical representation.)

All values of Ψ_B , Ψ_1 , Ψ_2 , $\Delta\varphi$ in degrees

$\alpha(\Psi_B)$



Crest R.F. Value = 48 KV

